Texas A&M University
Math Placement Exam for Math 142
Solutions to Practice Problems

1. $\left(\dfrac{14}{3+\sqrt{2}}\right)\left(\dfrac{3-\sqrt{2}}{3-\sqrt{2}}\right) = \dfrac{42-14\sqrt{2}}{9-2} = \dfrac{7(6-2\sqrt{2})}{7} = 6-2\sqrt{2}$

2. $\dfrac{x+2a-3}{x+a} - \dfrac{x+6}{2x} = \left(\dfrac{x+2a-3}{x+a}\right)\left(\dfrac{2x}{2x}\right) - \left(\dfrac{x+6}{2x}\right)\left(\dfrac{x+a}{x+a}\right) = \dfrac{(x+2a-3)(2x)-(x+6)(x+a)}{(x+a)(2x)} =$
$\dfrac{2x^2+4xa-6x-x^2-6x-xa-6a}{(x+a)(2x)} = \dfrac{x^2+3xa-12x-6a}{(x+a)(2x)}$

3. $\dfrac{6x^2+11xy-10y^2}{3x^2+10xy-8y^2} = \dfrac{(3x-2y)(2x+5y)}{(3x-2y)(x+4y)} = \dfrac{2x+5y}{x+4y}$

4. $5(x-7)-13(x-7)-6=0$
$(x-7)(5-13)-6=0$
$(x-7)(-8)-6=0$
$-8x+56-6=0$
$-8x+50=0$
$-8x=-50$
$x=50/8$

5. Multiple $-2x+4y=12$ by 3 and $3x-5y=-3$ by 2. This gives

$$\begin{aligned} -6x+12y &= 36 \\ 6x-10y &= -6. \end{aligned}$$

Add these two equations and we get $2y=30 \implies y=15$. Substituting $y=15$ into $3x-5y=-3$ gives $x=24$. So the point that satisfies both equations is $(24,15)$ and the value of $x+y$ is $24+15=39$.

6. Let x be the amount invested at $5\frac{1}{2}\%$ and y be the amount invested at $6\frac{3}{4}\%$. The resulting system of equations is

$$\begin{aligned} x+y &= 10,000 \\ 0.055x+0.0675y &= 650. \end{aligned}$$

Solve the system of equations to find x and y.
$x=10,000-y \implies 0.055(10,000-y)+0.0675y=650 \implies 550-0.055y+0.0675y=650 \implies 0.0125y=100 \implies y=8000$. Since $x+y=10,000$ we know $x=2000$ and thus \$6,000 more is invested in $6\frac{3}{4}\%$.

7. Since $2ax+3by=7c \implies 3by=7c-2ax \implies y=\dfrac{7c-2ax}{3b}=\dfrac{-2a}{3b}x+\dfrac{7c}{3b}$. If x decreases by 10 then we know $y=\dfrac{-2a}{3b}(x-10)+\dfrac{7c}{3b}=\dfrac{-2ax}{3b}+\dfrac{20a}{3b}+\dfrac{7c}{3b}$, so y will increase by $\dfrac{20a}{3b}$.

8. $\dfrac{8}{x+1}-\left(\dfrac{y}{z+2}\div\dfrac{y-4}{w}\right)=\dfrac{8}{x+1}-\left(\dfrac{y}{z+2}\cdot\dfrac{w}{y-4}\right)=\dfrac{8}{x+1}-\left(\dfrac{yw}{(z+2)(y-4)}\right)$
$=\dfrac{8(z+2)(y-4)-yw(x+1)}{(x+1)(z+2)(y-4)}=\dfrac{8zy-32z+16y-64-ywx-yw}{(x+1)(z+2)(y-4)}$

9. Using point slope equation we get $y - 1 = 7(x - 5) \implies y = 7x - 35 + 1 \implies y = 7x - 34$. Use this to find y when $x = -4$ gives $y = 7(-4) - 34 \implies y = -62$.

10. $\dfrac{4k - 6 - 16}{2k + 3 + 2} = 0 \implies \dfrac{4k - 22}{2k + 5} = 0 \implies 4k - 22 = 0 \implies k = 22/4 = 5.5$

11. $f(x) = -\sqrt{x + 2} + 7$

12. $x+2-(5x-10) \geq 3 \implies x+2-5x+10 \geq 3 \implies -4x+12 \geq 3 \implies -4x \geq -9 \implies x \leq 9/4$

13. $f(x) = \dfrac{x^2 - 3x - 2}{6x^2 - 54} = \dfrac{x^2 - 3x - 2}{6(x + 3)(x - 3)}$, so the domain is all real numbers such that $x + 3 \neq 0$ and $x - 3 \neq 0$. This gives the domain $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$.

14. For $\dfrac{2x^2 + 13}{(x + 1)(x - 1)}$ on $x < 0$ the domain is $(-\infty, -1) \cup (-1, 0)$. For the function $\dfrac{5x - 26}{x + 2}$ on $x \geq 0$ the domain is $[0, \infty)$. So, for the function $f(x)$ the domain is $(-\infty, -1) \cup (-1, \infty)$.

15. When $x = 0$ we get $2(0)+3y = 10 \implies y = 10/3$. When $y = 0$ we get $2x+3(0) = 10 \implies x = 5$. So the x-intercept is $(5, 0)$ and the y-intercept is $(0, 10/3)$.

16. $\left(\dfrac{2}{\sqrt{x^5}}\right)\left(\sqrt[3]{4x}\right) = \left(2x^{-5/2}\right)(4x)^{1/3}$

$= \left(2x^{-5/2}\right)(4^{1/3} \cdot x^{1/3}) = \left(2x^{-5/2}\right)\left((2^2)^{1/3}x^{1/3}\right) = \left(2x^{-5/2}\right)\left(2^{2/3}x^{1/3}\right) = \dfrac{2^{5/3}}{x^{13/6}}$

17. The transformed graph is $g(x) = \dfrac{1}{2}(x - 4)^2 + 10$.

18. $f \circ g = \dfrac{\frac{2}{x}}{\frac{2}{x} + 1} = \dfrac{\frac{2}{x}}{\frac{2}{x} + \frac{x}{x}} = \dfrac{\frac{2}{x}}{\frac{2+x}{x}} = \dfrac{2}{x} \cdot \dfrac{x}{2 + x} = \dfrac{2}{2 + x}$

19. (a) $[2, \infty)$
    (b) $[-4, 7)$
    (c) $(-\infty, -5)$
    (d) $(-\infty, \infty)$

20. (a) $0 \leq x < 2$
    (b) $x < 4$
    (c) $x \geq 7$

21. $\dfrac{f(2 + h) - f(2)}{h} = \dfrac{\sqrt{2 + h + 4} - \sqrt{2 + 4}}{h} = \left(\dfrac{\sqrt{6 + h} - \sqrt{6}}{h}\right) \cdot \left(\dfrac{\sqrt{6 + h} + \sqrt{6}}{\sqrt{6 + h} + \sqrt{6}}\right) = \dfrac{6 + h - 6}{h\left(\sqrt{6 + h} + \sqrt{6}\right)}$

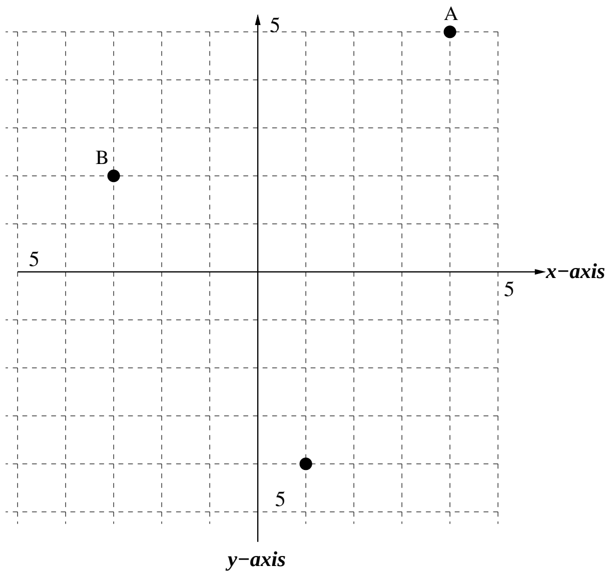
$= \dfrac{1}{\sqrt{6 + h} + \sqrt{6}}$

22. $\dfrac{x^2}{x^2 - x - 2} - \dfrac{4}{x^2 + x - 6} + \dfrac{x}{x^2 + 4x + 3} = \dfrac{x^2}{(x - 2)(x + 1)} - \dfrac{4}{(x + 3)(x - 2)} + \dfrac{x}{(x + 1)(x + 3)}$

$= \left(\dfrac{x + 3}{x + 3}\right)\left(\dfrac{x^2}{(x - 2)(x + 1)}\right) - \left(\dfrac{x + 1}{x + 1}\right)\left(\dfrac{4}{(x + 3)(x - 2)}\right) + \left(\dfrac{x - 2}{x - 2}\right)\left(\dfrac{x}{(x + 1)(x + 3)}\right)$

$\dfrac{x^3 + 3x^2 - 4x - 4 + x^2 - 2x}{(x + 3)(x - 2)(x + 1)} = \dfrac{x^3 + 4x^2 - 6x - 4}{(x + 3)(x - 2)(x + 1)}$

23. $f(2) = (2)^3 + 1 = 9$ and $f(-3) = 2(-3)^2 - 3 = 18 - 3 = 15$, so $f(2) - f(-3) = 9 - 15 = -6$.

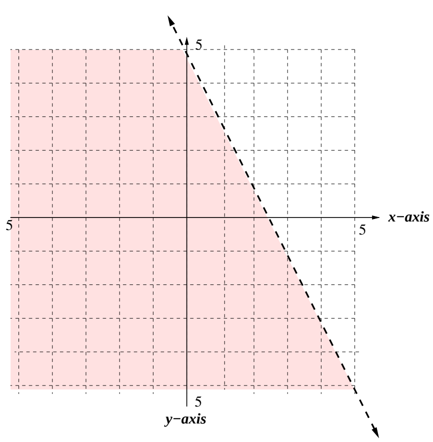
24. (a) 

(b) $y = 2$

(c) $x = 1$

(d) To find the slope we calculate $m = \dfrac{5 - -4}{4 - 1} = \dfrac{9}{3} = 3$. So in point-slope form the equation is $y - 5 = 3(x - 4)$.

(e) The slope of the line perpendicular to the line containing points A and C is $-\dfrac{1}{3}$. Using this slope with point B we get $y - 2 = -\dfrac{1}{3}(x + 3) \implies y = -\dfrac{1}{3}x + 1$.

25. 

Yes, the point $(-3, 8/3)$ lies in the solution set.

26. Simplify the following expressions

(a) $\dfrac{2x}{xy + xz + 5x} = \dfrac{2x}{x(y + z + 5)} = \dfrac{2}{(y + z + 5)}$

(b) $\dfrac{(2.4)(.4)}{(2.5)(.4) + (.6)(.4) + (1.9)(.4)} = \dfrac{(2.4)(.4)}{(.4)\left((2.5) + (.6) + (1.9)\right)} = \dfrac{2.4}{(2.5) + (.6) + (1.9)} = 0.48$

27. (a) $C(x) = 0.25x + 52.50$

(b) $R(x) = 2x$

(c) $P(x) = R(x) - C(x) = 2x - (0.25x + 52.50) = 2x - 0.25x - 52.50 = 1.75x - 52.50$

(d) $P(x) = 0$ when $1.75x - 52.50 = 0 \implies x = 52.50/1.75 = 30$. So Russ must sell 30 cups of lemondade to break even on his lemonade stand.

28. Apply the quadratic formula with $a = 3, b = 7$, and $c = -2$. This gives $x = \dfrac{-7 \pm \sqrt{7^2 - 4(3)(-2)}}{2(3)}$ $= \dfrac{-7 \pm \sqrt{49 + 24}}{6} = \dfrac{-7 \pm \sqrt{73}}{6}$. So one root is at $x = \dfrac{-7 + \sqrt{73}}{6}$ and the other root is at $x = \dfrac{-7 - \sqrt{73}}{6}$.

29. $\dfrac{8}{24} = \dfrac{1}{3}$

30. $7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5,040$

31. (a) $V = (10 - 2x)(6 - 2x)x = 4x^3 - 32x^2 + 60x$
    (b) $SA = 2x(6-2x)+2x(10-2x)+(6-2x)(10-2x) = 12x-4x^2+20x-4x^2+60-32x+4x^2 = -4x^2 + 60$

32. 31

33. 120

34. $\dfrac{66 \text{ feet}}{1 \text{ sec}} \cdot \dfrac{60 \text{ sec}}{1 \text{ min}} \cdot \dfrac{60 \text{ min}}{1 \text{ hour}} \cdot \dfrac{1 \text{ mile}}{5280 \text{ feet}} = 45\dfrac{\text{miles}}{\text{hour}}$