Texas A&M University
MPE2 Math Placement Exam
Solutions Practice Problems

1. $\left(\frac{14}{3+\sqrt{2}}\right)\left(\frac{3-\sqrt{2}}{3-\sqrt{2}}\right) = \frac{42-14\sqrt{2}}{9-2} = \frac{7(6-2\sqrt{2})}{7} = 6-2\sqrt{2}$

2. $\frac{x+2a-3}{x+a} - \frac{x+6}{2x} = \left(\frac{x+2a-3}{x+a}\right)\left(\frac{2x}{2x}\right) - \left(\frac{x+6}{2x}\right)\left(\frac{x+a}{x+a}\right) = \frac{(x+2a-3)(2x)-(x+6)(x+a)}{(x+a)(2x)} =$
$\frac{2x^2+4xa-6x-x^2-6x-xa-6a}{(x+a)(2x)} = \frac{x^2+3xa-12x-6a}{(x+a)(2x)}$

3. $\frac{6x^2+11xy-10y^2}{3x^2+10xy-8y^2} = \frac{(3x-2y)(2x+5y)}{(3x-2y)(x+4y)} = \frac{2x+5y}{x+4y}$

4. $5(x-7)-13(x-7)-6=0$
$(x-7)(5-13)-6=0$
$(x-7)(-8)-6=0$
$-8x+56-6=0$
$-8x+50=0$
$-8x=-50$
$x=50/8$

5. Multiple $-2x+4y=12$ by 3 and $3x-5y=-3$ by 2. This gives

$$-6x+12y=36$$
$$6x-10y=-6.$$

Add these two equations and we get $2y=30 \implies y=15$. Substituting $y=15$ into $3x-5y=-3$ gives $x=24$. So the point that satisfies both equations is $(24,15)$ and the value of $x+y$ is $24+15=39$.

6. Let x be the amount invested at $5\frac{1}{2}\%$ and y be the amount invested at $6\frac{3}{4}\%$. The resulting system of equations is

$$x+y=10,000$$
$$0.055x+0.0675y=650.$$

Solve the system of equations to find x and y.
$x=10,000-y \implies 0.055(10,000-y)+0.0675y=650 \implies 550-0.055y+0.0675y=650 \implies 0.0125y=100 \implies y=8000$. Since $x+y=10,000$ we know $x=2000$ and thus \$6,000 more is invested in $6\frac{3}{4}\%$.

7. Since $2ax+3by=7c \implies 3by=7c-2ax \implies y=\frac{7c-2ax}{3b}=\frac{-2a}{3b}x+\frac{7c}{3b}$. If x decreases by 10 then we know $y=\frac{-2a}{3b}(x-10)+\frac{7c}{3b}=\frac{-2ax}{3b}+\frac{20a}{3b}+\frac{7c}{3b}$, so y will increase by $\frac{20a}{3b}$.

8. $\frac{8}{x+1} - \left(\frac{y}{z+2} \div \frac{y-4}{w}\right) = \frac{8}{x+1} - \left(\frac{y}{z+2}\cdot\frac{w}{y-4}\right) = \frac{8}{x+1} - \left(\frac{yw}{(z+2)(y-4)}\right)$
$= \frac{8(z+2)(y-4)-yw(x+1)}{(x+1)(z+2)(y-4)} = \frac{8zy-32z+16y-64-ywx-yw}{(x+1)(z+2)(y-4)}$

9. Using point slope equation we get $y-1=7(x-5) \implies y=7x-35+1 \implies y=7x-34$. Use this to find y when $x=-4$ gives $y=7(-4)-34 \implies y=-62$.

10. $\dfrac{4k-6-16}{2k+3+2}=0 \implies \dfrac{4k-22}{2k+5}=0 \implies 4k-22=0 \implies k=22/4=5.5$

11. $f(x)=-\sqrt{x+2}+7$

12. $x+2-(5x-10)\geq 3 \implies x+2-5x+10\geq 3 \implies -4x+12\geq 3 \implies -4x\geq -9 \implies x\leq 9/4$

13. $f(x)=\dfrac{x^2-3x-2}{6x^2-54}=\dfrac{x^2-3x-2}{6(x+3)(x-3)}$, so the domain is all real numbers such that $x+3\neq 0$ and $x-3\neq 0$. This gives the domain $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$.

14. For $\dfrac{2x^2+13}{(x+1)(x-1)}$ on $x<0$ the domain is $(-\infty,-1)\cup(-1,0)$. For the function $\dfrac{5x-26}{x+2}$ on $x\geq 0$ the domain is $[0,\infty)$. So, for the function $f(x)$ the domain is $(-\infty,-1)\cup(-1,\infty)$.

15. When $x=0$ we get $2(0)+3y=10 \implies y=10/3$. When $y=0$ we get $2x+3(0)=10 \implies x=5$. So the x-intercept is $(5,0)$ and the y-intercept is $(0,10/3)$.

16. $\left(\dfrac{2}{\sqrt{x^5}}\right)\left(\sqrt[3]{4x}\right)=\left(2x^{-5/2}\right)(4x)^{1/3}$
$=\left(2x^{-5/2}\right)\left(4^{1/3}\cdot x^{1/3}\right)=\left(2x^{-5/2}\right)\left((2^2)^{1/3}x^{1/3}\right)=\left(2x^{-5/2}\right)\left(2^{2/3}x^{1/3}\right)=\dfrac{2^{5/3}}{x^{13/6}}$

17. The transformed graph is $g(x)=\dfrac{1}{2}(x-4)^2+10$.

18. $f\circ g=\dfrac{\frac{2}{x}}{\frac{2}{x}+1}=\dfrac{\frac{2}{x}}{\frac{2}{x}+\frac{x}{x}}=\dfrac{\frac{2}{x}}{\frac{2+x}{x}}=\dfrac{2}{x}\cdot\dfrac{x}{2+x}=\dfrac{2}{2+x}$

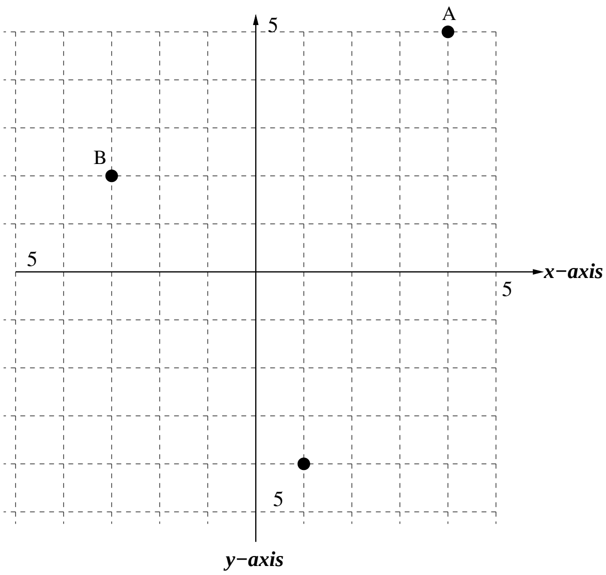
19. (a) $[2,\infty)$
    (b) $[-4,7)$
    (c) $(-\infty,-5)$
    (d) $(-\infty,\infty)$

20. (a) $0\leq x<2$
    (b) $x<4$
    (c) $x\geq 7$

21. $\dfrac{f(2+h)-f(2)}{h}=\dfrac{\sqrt{2+h+4}-\sqrt{2+4}}{h}=\left(\dfrac{\sqrt{6+h}-\sqrt{6}}{h}\right)\cdot\left(\dfrac{\sqrt{6+h}+\sqrt{6}}{\sqrt{6+h}+\sqrt{6}}\right)=\dfrac{6+h-6}{h\left(\sqrt{6+h}+\sqrt{6}\right)}$
$=\dfrac{1}{\sqrt{6+h}+\sqrt{6}}$

22. $\dfrac{x^2}{x^2-x-2}-\dfrac{4}{x^2+x-6}+\dfrac{x}{x^2+4x+3}=\dfrac{x^2}{(x-2)(x+1)}-\dfrac{4}{(x+3)(x-2)}+\dfrac{x}{(x+1)(x+3)}$
$=\left(\dfrac{x+3}{x+3}\right)\left(\dfrac{x^2}{(x-2)(x+1)}\right)-\left(\dfrac{x+1}{x+1}\right)\left(\dfrac{4}{(x+3)(x-2)}\right)+\left(\dfrac{x-2}{x-2}\right)\left(\dfrac{x}{(x+1)(x+3)}\right)$
$\dfrac{x^3+3x^2-4x-4+x^2-2x}{(x+3)(x-2)(x+1)}=\dfrac{x^3+4x^2-6x-4}{(x+3)(x-2)(x+1)}$

23. $f(2)=(2)^3+1=9$ and $f(-3)=2(-3)^2-3=18-3=15$, so $f(2)-f(-3)=9-15=-6$.

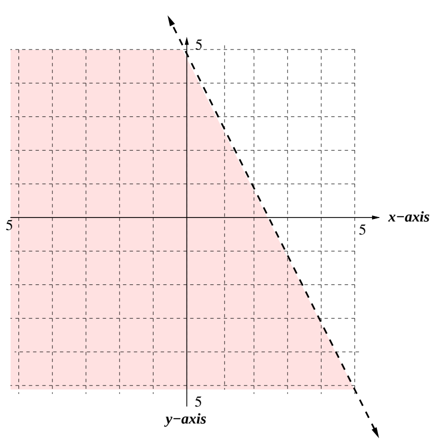
24. (a) 

(b) $y = 2$

(c) $x = 1$

(d) To find the slope we calculate $m = \dfrac{5 - -4}{4 - 1} = \dfrac{9}{3} = 3$. So in point-slope form the equation is $y - 5 = 3(x - 4)$.

(e) The slope of the line perpendicular to the line containing points A and C is $-\dfrac{1}{3}$. Using this slope with point B we get $y - 2 = -\dfrac{1}{3}(x + 3) \implies y = -\dfrac{1}{3}x + 1$.

25. 

Yes, the point $(-3, 8/3)$ lies in the solution set.

26. Simplify the following expressions

(a) $\dfrac{2x}{xy + xz + 5x} = \dfrac{2x}{x(y + z + 5)} = \dfrac{2}{(y + z + 5)}$

(b) $\dfrac{(2.4)(.4)}{(2.5)(.4) + (.6)(.4) + (1.9)(.4)} = \dfrac{(2.4)(.4)}{(.4)\left((2.5) + (.6) + (1.9)\right)} = \dfrac{2.4}{(2.5) + (.6) + (1.9)} = 0.48$

27. (a) $C(x) = 0.25x + 52.50$

(b) $R(x) = 2x$

(c) $P(x) = R(x) - C(x) = 2x - (0.25x + 52.50) = 2x - 0.25x - 52.50 = 1.75x - 52.50$

(d) $P(x) = 0$ when $1.75x - 52.50 = 0 \implies x = 52.50/1.75 = 30$. So Russ must sell 30 cups of lemondade to break even on his lemonade stand.

28. Apply the quadratic formula with $a = 3, b = 7$, and $c = -2$. This gives $x = \frac{-7 \pm \sqrt{7^2 - 4(3)(-2)}}{2(3)}$ $= \frac{-7 \pm \sqrt{49 + 24}}{6} = \frac{-7 \pm \sqrt{73}}{6}$. So one root is at $x = \frac{-7 + \sqrt{73}}{6}$ and the other root is at $x = \frac{-7 - \sqrt{73}}{6}$.

29. $\frac{8}{24} = \frac{1}{3}$

30. $7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5,040$

31. (a) $V = (10 - 2x)(6 - 2x)x = 4x^3 - 32x^2 + 60x$
    (b) $SA = 2x(6-2x)+2x(10-2x)+(6-2x)(10-2x) = 12x-4x^2+20x-4x^2+60-32x+4x^2 = -4x^2 + 60$

32. 31

33. 120

34. $\frac{\text{66 feet}}{\text{1 sec}} \cdot \frac{\text{60 sec}}{\text{1 min}} \cdot \frac{\text{60 min}}{\text{1 hour}} \cdot \frac{\text{1 mile}}{\text{5280 feet}} = 45 \frac{\text{miles}}{\text{hour}}$